## Diskussionsbeitrag / Discussion Paper

# Coin Migration and Seigniorage within the Euro Area

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### Summary

The introduction of the euro coins in 2002 presented a unique opportunity to study the cross-border migration and the mixing process of coins in different euro-area countries. This note analyses how many euro coins flow from and into Germany and which composition of coins is to be expected in the long run. A simple model of coin migration is formulated and calibrated for €1 coins. The ratio of German coins circulating in Germany is likely to settle around 50 % if past growth of coin volumes continues. In contrast to banknotes, allocation of coin seigniorage depends on national coin issuance. Different national growth rates of coin volumes have important implications for coin seigniorage.

## 1 Introduction

Since January 1, 2002, euro banknotes and coins have been legal tender in the euro area. In contrast to banknotes, it is easy to recognize the origin of the various euro area coins owing to their unique national backsides. As people clearly notice in their everyday cash transactions, the national euro coins have become mixed over time. The introduction of euro coins in 2002 presented a unique opportunity to study the cross-border migration of euro coins and the mixing of coins. This mixing process of coins within a currency area is not only interesting from a theoretical point of view or in a statistical sense. It has also important political consequences since seigniorage revenues accruing to the EMU countries depend on national coin issues.

This note is confined to Germany and €1 coins.[1] For this case, consistent data from the *„Euromobil"* and *„Eurodiff"* projects are available. We focus on the 'indirect' calcula-

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[1] For smaller denominations, the results would be distorted owing to a considerable amount of lost coins, whereas in the case of €2 coins special-issue coins would have to be considered as well. In the US, it was even observed up until recently that the demand for coins went up perceptibly in the wake of surging commodity prices, creating an incentive to melt coins (Velde 2007). Melting has since been prohibited by the US Treasury and does not seem to be an issue for euro coins owing to their different metallic composition.

tion of the coin migration process based on a simple model. Following on from this, we estimate the share of German coins circulating in Germany to be expected in the long term. Section 2 outlines the model and discusses the results. In section 3, the implications of nationally different growth rates of coin issuance for coin seigniorage are considered. Section 4 draws some conclusions.

## 2 Coin migration model

There are few projects investigating the coin diffusion process. A French project is based on representative surveys (Grasland et al. 2002), whereas other projects largely rely on voluntary reporting. Examples of the latter are the *„Euromobil“* and *„Eurodiff“* projects in Germany, which focus exclusively on €1 coins (Stoyan 2002; Stoyan et al. 2004), *„Eurotracer“*, which also includes the migration of banknotes, and *„Eurodiffusie“* in the Netherlands and Belgium (Blokland et al. 2002).[2] Furthermore, since 2002, the European Commission has been taking random samples of coins from major towns and cities at irregular intervals to gather data on the coin mixing process.

Table 1 provides the distribution of €1 coins in circulation in the euro-area. At the end of 2008 1,340 million coins were of German origin and 4,660 million were coins issued from other member states. This corresponds to approximately 16 €1 coins per capita in Germany and 20 €1 coins per capita in the rest of the euro area. Statistical research carried out by Stoyan suggests that, at the end of 2008, around 75 % of the €1 coins circulating in Germany at that time were of German origin.[3] That means, an estimated total of 335 million German €1 coins have already migrated abroad, which is equivalent to around 48 million coins per year, or 130,000 coins per day.[4] We assume that in the same period 335 million foreign coins flowed into Germany, as we have no evidence to the contrary (Deutsche Bundesbank 2003: 208).

In the following we outline a simple model of the coin migration process for €1 coins.[5] Abstracting from leakages and outflows to non-EMU countries, German coins at time t ($N_D(t)$), are either circulating in Germany ($N_{DD}(t)$) or they have migrated to other euro-area countries ($N_{DA}(t)$):

$$N_D(t) = N_{DD}(t) + N_{DA}(t). \tag{1}$$

As a result of foreign travel (tourism, business trips) a certain amount of German €1 coins drifts to other euro-area countries. We assume that this amount is proportional to the volume at the start of the period: $\alpha N_{DD}(t\text{-}1)$. Conversely, coins also flow from other mem-

[2] See http://www.mathe.tu-freiberg.de/inst/stoch/Stoyan/euro/euro.html and http://www.eurotracer.net.

[3] See http://www.mathe.tu-freiberg.de/Stoyan/euro/en/euro.html

[4] According to information from Fraport AG, 7.2 million passengers from other euro area countries landed at Frankfurt International Airport in 2007. This amounts statistically to around 20,000 passengers per day. Frankfurt is the biggest German airport which handles around one-third of all air passengers in Germany. Moreover, other means of transport are also used. And commuters travelling to and from Germany on a daily basis also have to be taken into account. In 2006, a (net) daily average of 90,000 persons commuted to Germany. Most of them are likely to come from other euro-area countries, primarily Austria and the Netherlands (Stoyan et al. 2004: 74). Therefore, the flow rates – about 130,000 per day – appear to be plausible.

[5] For a more detailed discussion and different specifications as well as a formulation of the model in terms of Markov chains, see Seitz et al. (2009).

**Table 1** €1 coin volumes at the end of 2008 (in millions)

| | in Germany | Abroad | Total |
|---|---|---|---|
| German coins | $N_{DD}$ = 1005 | $N_{DA}$ = 335 | $N_D$ = 1340 |
| Foreign coins | $N_{AD}$ = 335 | $N_{AA}$ = 4325 | $N_A$ = 4660 |
| | | | N = 6000 |

*Source:* Deutsche Bundesbank, European Central Bank and Stoyan (see footnote 3)

ber states to Germany. This includes German coins which already migrated abroad in previous periods. Here, too, we assume that this amount is proportional to the number of coins abroad at the beginning of the period: ß$N_{DA}$(t-1). Moreover, in every period new coins are issued in Germany as well as in the other member states.[6] Assuming constant growth rates ($g_D$, $g_A$), we get the following equation for the evolution of the number of German €1 coins circulating in Germany:

$$N_{DD}(t) = N_{DD}(t-1) - \alpha N_{DD}(t-1) + \beta N_{DA}(t-1) + g_D N_D(t-1). \tag{2}$$

The final term is the number of German €1 coins newly issued in period t. Since we are primarily interested in the share of coins circulating at home and abroad, we define the share of German coins in Germany as $n_{DD}(t) \equiv N_{DD}(t)/N_D(t)$and rewrite (2) as:

$$n_{DD}(t) = \frac{\beta + g_D}{1 + g_D} + \frac{1 - \alpha - \beta}{1 + g_D} n_{DD}(t-1). \tag{3}$$

At the time of the first issue (end of 2001), all German coins were located in Germany such that the starting value is $n_{DD}(0) = 1$. In the long run, the share of German coins in Germany converges towards

$$n^*_{DD} = \frac{\beta + g_D}{\alpha + \beta + g_D}. \tag{4}$$

The greater the backflow rate together with the growth rate (ß+$g_D$) and the smaller the outflow rate ($\alpha$), the greater is the share of German coins that will circulate in Germany in the long run (and therefore the smaller the share of German coins abroad). If no outflow occurs ($\alpha$=0), the share of German coins in Germany remains at $n_{DD}(0) = n^*_{DD} = 1$. Conversely, if no backflow occurs and no new coins are issued (ß = $g_D$ = 0), all German €1 coins eventually migrate abroad ($n^*_{DD} = 0$). From (3) we obtain the following equation for the evolution of coin shares over time :

$$n_{DD}(t) = n^*_{DD} + \left(\frac{1 - \alpha - \beta}{1 + g_D}\right)^t \left(1 - n^*_{DD}\right). \tag{5}$$

According to this dynamic adjustment process, the share of German coins in Germany decreases monotonically and converges towards its equilibrium value $n^*_{DD}$. The larger the outflow rate ($\alpha$) and the backflow rate (ß), the quicker the coins are mixed. On the other

[6] Between 2002 and 2008, the volume of German €1 coins on average grew by almost 4 % per year and the volume of foreign coins by about 10 %.The latter figure might be slightly overstated because it includes the new euro area members Slowenia, Malta and Cyprus.

hand, a large growth rate ($g_D$) slows down the mixing process. An analogous equation holds for the share of foreign €1 coins abroad ($n_{AA}$),

$$n_{AA}(t) = n^*_{AA} + \left(\frac{1-\alpha-\beta}{1+g_A}\right)^t (1 - n^*_{AA}), \tag{6}$$

where

$$n^*_{AA} = \frac{\alpha + g_A}{\alpha+\beta+g_A}. \tag{7}$$

Equations (5) and (6) can be solved numerically for the parameters $\alpha$ and $\beta$. We use the calibration $n_{DD}(7) = 1005/1340 = 0.75$, $n_{AA}(7) = 4325/4660 = 0.928$ from Table 1 and the average growth rates, $g_D = 4\,\%$ and $g_A = 10\,\%$, observed in the past. The resulting estimates for the flow parameters as well as the implied long run equilibrium coin shares are shown in Table 2 (row a).

According to these estimates, annually 5.0 % of German €1 coins flow to other euro-area countries and 1.8 % of foreign €1 coins flow to Germany. This implies that in the long run the share of German €1 coins in Germany would fall from presently about 75 % to 53.3 %, whereas the ratio of foreign coins circulating abroad would merely fall from 92.8 % to 89.5 %.

Alternatively, assuming equal growth rates of coin volumes from 2009 onwards ($g_D$=$g_A$=0.04), the equilibrium share of Germany would not be affected as the assumed growth rate is still 4 %. However, for the other EMU countries it would drop from 89.5 to 83,7 % (row b). If the coin volumes remained constant from 2009 onwards ($g_D = g_A = 0$), the estimated equilibrium shares would sharply fall to 25.9 % for Germany and to 74.1 % for the other EMU countries, respectively (row c). Figure 1 shows the estimated evolution of German coins in Germany (lower lines) and of foreign coins abroad (upper lines).

In the calibration period 2008 (t=7) the number of German coins circulating abroad matches the number of foreign coins in Germany ($N_{DA}(7) = N_{AD}(7) = 335$ mill.). Under this assumption and for balanced growth ($g_D = g_A = g$), our model implies that $N_{DA}(t) = N_{AD}(t)$ will continue to hold in subsequent periods. However, if the national growth rates of coin issuance differ (as they did in the past), the country with the larger growth rate imposes its coins on the country with the smaller growth rate, which becomes a net importer of foreign coins. Thus, the number of coins circulating within a country starts to diverge from the number of coins issued by that country. Specifically, the number of coins

**Table 2** Estimated flow rates and equilibrium coin shares

| **Estimated …** | **Parameters** | | **Equilibrium shares** | |
|---|---|---|---|---|
| | $\hat{\alpha}$ | $\hat{\beta}$ | $\hat{n}^*_{DD}$ | $\hat{n}^*_{AA}$ |
| (a) Observed growth * | 0.0504 | 0.0176 | 0.533 | 0.895 |
| (b) Equal growth ** | " | " | 0.533 | 0.837 |
| (c) No growth *** | " | " | 0.259 | 0.741 |

* Observed growth rates: $g_D = 4\,\%$, $g_A = 10\,\%$;
Assumed growth rates from 2009 onwards: ** $g_D = g_A = 4\,\%$; *** $g_D = g_A = 0\,\%$

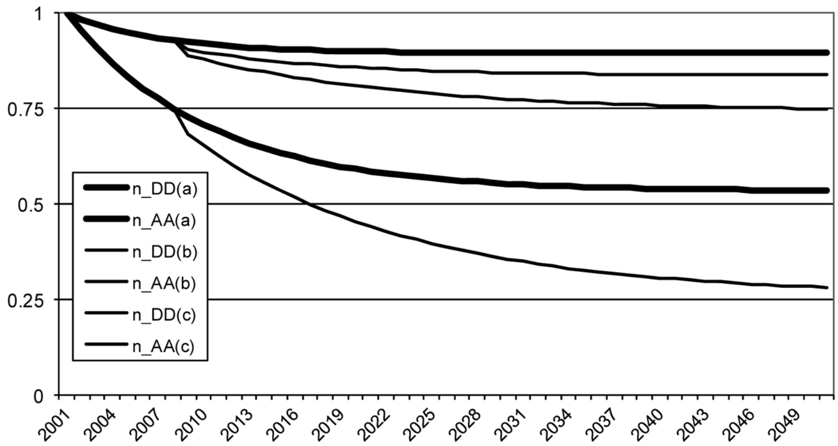


**Figure 1** Evolution of coin shares

circulating in Germany ($H_D$) is the sum of the number of coins issued ($N_D$) plus the net imports to Germany:

$$H_D(t) \equiv N_D(t) + [N_{AD}(t) - N_{DA}(t)] = N_{AD}(t) + N_{DD}(t) \tag{8}$$

Let

$$\psi(t) \equiv \frac{N_A(t)}{N_D(t)} = \frac{N_A(0)}{N_D(0)} \left( \frac{1+g_A}{1+g_D} \right)^t \tag{9}$$

denote the ratio of €1 coins issued abroad to those issued by Germany. This expression converges to a constant in the long run only for balanced growth ($g_A = g_D$). In all other cases, however, $\psi$(t) either tends to zero (if $g_A < g_D$) or to infinity (if $g_A > g_D$). Using (9), the number of coins circulating in Germany can be written as:

$$H_D(t) = N_A(t) \left[ n_{AD}(t) + \frac{n_{DD}(t)}{\psi(t)} \right] \tag{8'}$$

If $g_A > g_D$, the final term in (8') vanishes and, since $n_{AD}$(t) converges to a constant, the number of coins circulating in Germany becomes directly proportional to the number of coins issued abroad ($N_A$). Thus, the growth rate of coins circulating in Germany ($H_D$) converges towards the larger growth rate $g_A$ ($> g_D$).[7] Obviously, such a scenario cannot prevail forever. Although different growth rates of coin issuance are not sustainable in the long run, they may have important consequences for coin seigniorage for a prolonged transitional period of time.

[7] This result also implies that the ratio of German coins circulating in Germany will converge to zero.

## 3 Implications for coin seigniorage

According to Article 106(2) of the EU Treaty, production and issuance of euro coins is largely organised along national lines. In the euro area the member countries have the right to determine the amounts of coins to be issued. In Germany, this is done by the Ministry of Finance in coordination with the Deutsche Bundesbank. Furthermore, the amounts have to be approved by the ECB. Except this approval, there is no upper limit to the issuance process. However, according to the EU Treaty, the maximum amount of coins in the stocks of the national central banks to be credited to the government must not exceed 10 % of the total issue of the respective country. Therefore, if the cross-border coin flows are not symmetric or national coin demands differ, this has consequences for the seigniorage revenues of the different countries.

Coins are a direct source of seigniorage which accrues to the finance ministries of the member states. As will be explained below, there is no pooling as with euro banknotes. It is customary in most euro area countries to consider national seigniorage from euro coins as corresponding to the face value less production costs.[8] The estimated average *variable* production costs vary from 6.2 % (€2) to 90 % (€0.01) of face value. For €1 coins variable production costs are 10.4 % on average, with a range between 5.6 % and 15.3 %. Table 3 shows the estimated seigniorage income on euro coins of Germany and the other member states in 2002 and over the period 2003 to 2007 based on average variable production costs. €1 coins account for roughly one third of the total value of coin issuance in the euro area.

**Table 3** Estimated seigniorage on euro coins (in 2002-07, mill. €)

| | **2002** | **2003-07** | **Total** | **%** |
|---|---|---|---|---|
| Germany | 3,179 | 1,566 | 4,745 | *28.31* |
| Others | 7,707 | 4,310 | 12,017 | *71.69* |
| Euro Area | 10,886 | 5,876 | 16,762 | *100* |

*Source:* EFC (2008: 103)

What are the implications of different growth rates of national coin issuance for seigniorage on €1 coins? Assuming for simplicity that variable production costs (c) are the same in the member states (D, A), the *cumulated* amount of seigniorage on €1 coins in Germany is

$$S_D(t) = (1 - c)\, N_D(t) \tag{10}$$

and the total amount of seigniorage collected in the euro-area is

$$S(t) = (1 - c)[N_D(t) + N_A(t)]. \tag{11}$$

[8] This measure is usually used for the calculation of seigniorage earned by the Treasury from coin issue. Different seigniorage concepts are discussed by Baltensperger and Jordan (1997).

If in both countries coin issues grow at constant, but possibly different, rates ($g_A$, $g_D$), the (*cumulated*) share of German seigniorage on €1 coins is:

$$s_D(t) \equiv \frac{S_D(t)}{S(t)} = \frac{N_D(t)}{N_D(t) + N_A(t)} = \frac{1}{1 + \psi(t)} \tag{12}$$

where $\psi$(t) is defined as in (9). Only in the special case of balanced growth rates of coin issuance ($g_A = g_D$), the German share converges to a constant.[9] However, if $g_A > g_D$ (as has been the case up to 2009) and thus $\psi \to \infty$, the German seigniorage share decreases gradually and eventually approaches zero.[10]

To avoid this undesirable result, other schemes of seigniorage allocation among the member states must be considered. One obvious scheme is to use the same allocation key as for euro banknotes. The distribution of seigniorage from euro banknotes does not depend on the volumes issued by each national central bank (NCB). Rather, seigniorage is pooled and subsequently redistributed among the NCB's proportional to the ECB capital key, which is based on GDP and population. Adopting this allocation scheme, German seigniorage becomes

$$S_D(t) = k_D \cdot S(t) \tag{13}$$

where $k_D$ denotes the German capital share. Regardless of national growth rates, the national shares in coin seigniorage would remain constant as long as the capital shares are fixed. In t=0, both allocation rules, (10) and (13), yield the same distribution of seigniorage from €1 coins if the German capital share is equal to the German share of coins issued $k_D = N_D(0)/[N_D(0) + N_A(0)]$. In subsequent periods, both rules would continue to yield the same seigniorage distribution if the growth rates of coin issuance were the same.

Table 4 provides hypothetical calculations based on allocation rules (10) and (13), respectively. Both rules result in almost the same seigniorage distribution initially (end of 2001). After seven years (at the end of 2008), the current allocation rule produces a seigniorage loss for Germany on €1 coins of about 400 mill € compared to the rule based on capital keys. If the growth rates of coin volumes issued ($g_D$ = 4 %, $g_A$ = 10 %) continue for another seven years (to the end of 2015), the estimated seigniorage loss of Germany would accumulate to almost 1.3 billion €. Therefore, the present allocation scheme favours those countries which record a net outflow of coins.[11] These countries benefit, as they issue more coins than are actually required domestically.

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9 To be specific, the German share converges towards ß/($\alpha$+ß).

10 In the latter case Germany would experience an increasing amount of net inflows of foreign coins. If area-wide coin issuance grows faster than coin demand, one would see the build-up of surplus stocks and, eventually, net back flows (negative net issuance), even possibly resulting in negative seigniorage.

11 Historically, this kind of situation occurred time and time again and, in some cases, led to a coin shortage. A common response to this in the past was to impose a ban on coin exports. A theoretical analysis of this relationship can be found in Wallace and Zhou (1997).

**Table 4** Allocation of seigniorage on €1 coins (mill €)

| | Germany | Other EMU-countries | Total |
|---|---|---|---|
| | | (a) Allocation based on national issuance* | |
| End of 2001 (t=0) | 912 | 2,143 | 3,055 |
| 2002–2008 (t=7) | 1,201 | 4,175 | 5,376 |
| 2002–2015 (t=14) | 1,580 | 8,137 | 9,717 |
| | | (b) Allocation based on capital key** | |
| End of 2001 | 902 | 2,153 | 3,055 |
| 2002–2008 | 1,587 | 3,789 | 5,376 |
| 2002–2015 | 2,869 | 6,848 | 9,717 |
| | | (c) Differences (a-b) | |
| End of 2001 | 10 | –10 | 0 |
| 2002–2008 | –387 | 387 | 0 |
| 2002–2015 | –1,289 | 1,289 | 0 |

** Equation (10), $c = 0.104$; ** Equation (13), capital key for Germany as of 2007: $k_D = 0.295229$.*

## 4 Conclusions

This note captures the mixing process of euro coins and determines the evolution of the share of German euro coins in Germany by means of a simple model, calibrated with data on €1 coins in Germany. According to our calculations, a total of about 5 % of German coins flow abroad each year. If coin volumes continue to grow at rates observed in the past, the share of German €1 coins circulating in Germany is likely to remain above 50 %. However, different national growth rates of coin issuance create problems with the allocation of coin seigniorage. The present allocation scheme, which is based on national issuance, favours countries that realize above average growth rates. Pooling coin seigniorage on the basis of ECB capital keys, as it is the case for banknotes, would avoid incentives of excess supply. Interestingly, as Sinn and Feist (2000) have shown, Germany has lost seigniorage on banknotes with the introduction of the euro in 1999 due to this pooling agreement.

## References


Baltensperger, E., T. J. Jordan (1997), Principles of Seigniorage. Swiss Journal of Economics and Statistics 133: 133-151.

Blokland, P. van, L. Booth, K. Hiremath, M. Hochstenbach, G. Koole, S. Pop, M. Quant, D. Wirosoetisno (2002), The Euro Diffusion Project. Proceedings of the 42nd European Study Group with Industry 42: 41-57.

Deutsche Bundesbank (2003), Münzgeldentwicklung in Deutschland. Frankfurt a. M.

EFC (2008), Final Report from the Task Force on the Consequences of Net Euro Coin Migration. Economic and Financial Committee, Brussels, 30 June.

Grasland, C., F. Guérin-Pace, A. Tostain (2002), The Circulation of Euros as a Reflection of People's Mobility. Population and Societies 384, November, 2-4.

Seitz, F., D. Stoyan, K.-H. Tödter (2009), Coin Migration within the Euro Area. Deutsche Bundesbank, Discussion Paper, Series 1, No. 27/2009.

Sinn, H.-W., H. Feist (2000), Seignorage Wealth in the Eurosystem: Eurowinners and Eurolosers Revisited. CESifo Working Paper No. 353, November.

Stoyan, D. (2002), Statistical Analyses of Euro Coin Mixing. Mathematical Spectrum 35: 50-55.

Stoyan, D., H. Stoyan, G. Döge (2004), Statistical Analyses and Modelling of the Mixing Process of Euro Coins in Germany and Europe, Australian & New Zealand. Journal of Statistics 46: 67-77.

Velde, F. R. (2007), What's a Penny (or a Nickel) Really Worth? Federal Reserve Bank of Chicago, Chicago Fed letter no. 235a, February.

Wallace, N., R. Zhou (1997), A Model of Currency Shortage. Journal of Monetary Economics 40: 555-572.

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