2.1 a) $(f\circ g)(x) = \sqrt{\ln x} \qquad D_f = [1, \infty[$

b) $(f\circ g)(x) = \ln \sqrt{x} \qquad D_f =]0, \infty[$

c) $((f\circ g)\circ h)(x) = \sqrt{\ln \frac{1}{x}} = \sqrt{-\ln x} \qquad D_f =]0, 1]$

d) $(g\circ(f\circ h))(x) = \ln \sqrt{\frac{1}{x}} \qquad D_f =]0, \infty[$

2.2

$$f(x) = \sqrt{1+x^2} \longrightarrow \sqrt{1+(x-4)^2} \longrightarrow \sqrt{1+(\tfrac{1}{2}x-4)^2}$$

$$\longrightarrow 3\sqrt{1+(\tfrac{1}{2}x-4)^2} \longrightarrow 3\sqrt{1+(\tfrac{1}{2}x-4)^2} - 2 = \tilde{f}(x)$$

2.3 a)

$$4x^2-24x+31 = 4(x^2-6x)+31 = 4(x^2-6x+9-9)+31$$

$$= 4(x^2-6x+9)-36+31 = 4(x-3)^2-5$$

$$= (2(x-3))^2-5 = (2x-6)^2-5$$

$$x^2 \rightarrow 4x^2 \longrightarrow 4(x-3)^2 \longrightarrow 4(x-3)^2-5$$

oder

$$x^2 \rightarrow (x-6)^2 \longrightarrow (2x-6)^2 \rightarrow (2x-6)^2-5$$

b)

$$ax^2+bx+c = a(x^2+\tfrac{b}{a}x)+c$$

$$= a\left(x^2+2\tfrac{b}{2a}x + (\tfrac{b}{2a})^2 - (\tfrac{b}{2a})^2\right) + c$$

$$= a\left(x^2+2\tfrac{b}{2a}x+(\tfrac{b}{2a})^2\right) + c - \frac{b^2}{4a}$$

$$= a\left(x+\tfrac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$x^2 \rightarrow ax^2 \longrightarrow a(x+\tfrac{b}{2a})^2 \longrightarrow a(x+\tfrac{b}{2a})^2 + c - \frac{b^2}{4a}$$

c)

$$x^3+3x^2+3x+2 = x^3+3x^2+3x+1+1 = (x+1)^3+1$$

$$x^3 \rightarrow (x+1)^3 \longrightarrow (x+1)^3+1$$

d) $f(x) \to f(x+b) \to f(ax+b) \to c\,f(ax+b) \to c\,f(ax+b)+d$

$f(x) \to f(ax) \to f(a(x+\frac{b}{a})) = f(ax+b)$

$\to c\,f(ax+b) \to c\,f(ax+b)+d$

2.4 a) $x^4-1 = (x^2-1)(x^2+1) = (x-1)(x+1)(x^2+1)$

$$\frac{3x^3+3x+2}{x^4-1} = \frac{3x^3+3x+2}{(x-1)(x+1)(x^2+1)}$$

$$= \frac{A_{11}}{x-1} + \frac{A_{21}}{x+1} + \frac{B_{11}x+C_{11}}{x^2+1}$$

$$= \frac{A_{11}(x+1)(x^2+1)}{(x-1)(x+1)(x^2+1)} + \frac{A_{21}(x-1)(x^2+1)}{(x-1)(x+1)(x^2+1)} + \frac{(B_{11}x+C_{11})(x-1)(x+1)}{(x-1)(x+1)(x^2+1)}$$

$$= \frac{A_{11}(x+1)(x^2+1) + A_{21}(x-1)(x^2+1) + (B_{11}x+C_{11})(x-1)(x+1)}{(x-1)(x+1)(x^2+1)}$$

$$\frac{(A_{11}+A_{21}+B_{11})x^3 + (A_{11}-A_{21}+C_{11})x^2 + (A_{11}+A_{21}-B_{11})x + (A_{11}-A_{21}-\underline{C_{11}})}{x^4-1}$$

$$= \frac{3x^3+3x+2}{x^4-1}$$

$$\begin{aligned} A_{11} + A_{21} + B_{11} \quad &= 3 \\ A_{11} - A_{21} \quad + C_{11} &= 0 \\ A_{11} + A_{21} - B_{11} \quad &= 3 \\ A_{11} - A_{21} \quad - C_{11} &= 2 \end{aligned}$$

Lösung: $A_{11} = 2 \quad A_{21} = 1 \quad B_{11} = 0 \quad C_{11} = -1$

$$\Rightarrow f(x) = \frac{2}{x-1} + \frac{1}{x+1} - \frac{1}{x^2+1}$$

b)

$$x^4-2x^3+2x^2-2x+1 = x^4-4x^3+6x^2-4x+1 \quad +2x^3-4x^2+2x$$

$$= (x-1)^4+2x(x^2-2x+1) = (x-1)^4+2x(x-1)^2$$

$$= (x-1)^2\left[(x-1)^2+2x\right] = (x-1)^2(x^2-2x+1+2x) = (x-1)^2(x^2+1)$$

$$\frac{2x^2+2x+2}{x^4-2x^3+2x^2-2x+1} = \frac{2x^2+2x+2}{(x-1)^2(x^2+1)} = \frac{A_{11}}{x-1}+\frac{A_{12}}{(x-1)^2}+\frac{B_{11}x+C_{11}}{x^2+1}$$

$$= \frac{A_{11}(x-1)(x^2+1)+A_{12}(x^2+1)+(B_{11}x+C_{11})(x-1)^2}{(x-1)^2(x^2+1)}$$

$$= \frac{(A_{11}+B_{11})x^3+(-A_{11}+A_{12}-2B_{11}+C_{11})x^2+(A_{11}+B_{11}-2C_{11})x+(-A_{11}+A_{12}+C_{11})}{(x-1)^2(x^2+1)}$$

$$= \frac{2x^2+2x+2}{(x-1)^2(x^2+1)}$$

$$\begin{aligned}
A_{11} \qquad\quad + B_{11} \qquad\quad &= 0\\
-A_{11}+A_{12}-2B_{11}+C_{11} &= 2\\
A_{11} \qquad\quad + B_{11}-2C_{11} &= 2\\
-A_{11}+A_{12} \qquad\quad + C_{11} &= 2
\end{aligned}$$

Lösung: $A_{11}=0 \quad A_{12}=3 \quad B_{11}=0 \quad C_{11}=-1$

$$\Rightarrow f(x) = \frac{3}{(x-1)^2}-\frac{1}{x^2+1}$$

2.5

$\lg x = \frac{\ln x}{\ln 10} \qquad \ln x = \ln 10 \cdot \lg x \qquad$ Faktor $\ln 10$

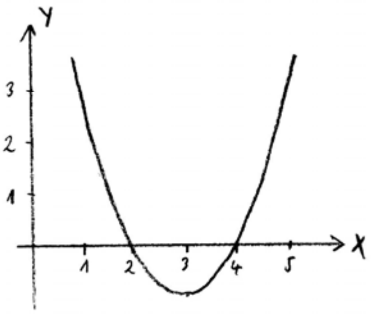
Graph von $\ln x$ entsteht aus Graph von $\lg x$ durch vertikale Dehnung
Graph von $\lg x$ entsteht aus Graph von $\ln x$ durch vertikale Stauchung

2.6

$10^x = e^{x \ln 10} \qquad$ Faktor $\ln 10$

Graph von 10^x entsteht aus Graph von e^x durch horizontale Stauchung
Graph von e^x entsteht aus Graph von 10^x durch horizontale Dehnung

2.7 a) $f(x) = x^2 - 6x + 8 = x^2 - 2\cdot 3x + 9 - 1 = (x-3)^2 - 1$



umkehrbar mit $D_f = [3,\infty[$

$W_f = [-1,\infty[$

$y = (x-3)^2 - 1$

$(x-3)^2 = y+1$

$x - 3 = \sqrt{y+1}$

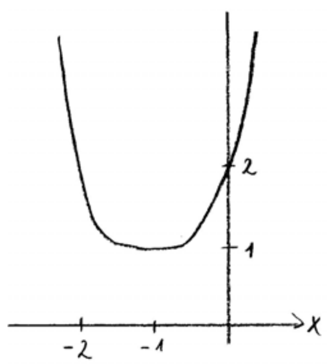
$x = \sqrt{y+1} + 3 = f^{-1}(y)$

$f^{-1}(x) = \sqrt{x+1} + 3$

$D_{f^{-1}} = [-1,\infty[\quad W_{f^{-1}} = [3,\infty[$

b) $f(x) = x^4 + 4x^3 + 6x^2 + 4x + 2 = x^4 + 4x^3 + 6x^2 + 4x + 1 + 1$

$= (x+1)^4 + 1$



umkehrbar mit $D_f = [-1,\infty[$

$W_f = [1,\infty[$

$y = (x+1)^4 + 1$

$(x+1)^4 = y - 1$

$x+1 = \sqrt[4]{y-1}$

$x = \sqrt[4]{y-1} - 1 = f^{-1}(y)$

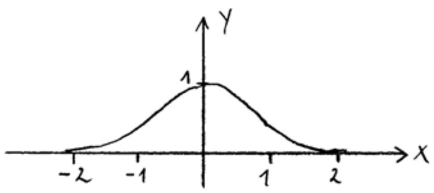
$f^{-1}(x) = \sqrt[4]{x-1} - 1$

$D_{f^{-1}} = [1,\infty[\quad W_{f^{-1}} = [-1,\infty[$

c) $f(x) = e^{-x^2}$

umkehrbar mit $D_f = [0,\infty[$

$W_f =]0,1]$



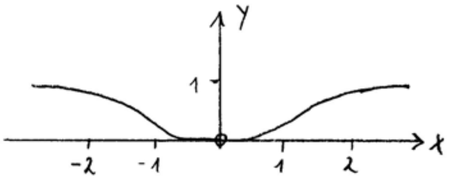
$y = e^{-x^2} \quad \ln y = -x^2$

$x^2 = -\ln y \quad x = \sqrt{-\ln y} = f^{-1}(y)$

$f^{-1}(x) = \sqrt{-\ln x} \quad D_{f^{-1}} =]0,1] \quad W_{f^{-1}} = [0,\infty[$

d)

$$f(x) = e^{-\frac{1}{x^2}}$$



umkehrbar mit $D_f =]0,\infty[$

$W_f =]0,1[$

$$y = e^{-\frac{1}{x^2}} \qquad \ln y = -\frac{1}{x^2}$$

$$x^2 = -\frac{1}{\ln y} \qquad x = \sqrt{-\frac{1}{\ln y}} = f^{-1}(y)$$

$$f^{-1}(x) = \sqrt{-\frac{1}{\ln x}} \qquad D_{f^{-1}} =]0,1[\qquad W_{f^{-1}} =]0,\infty[$$

2.8 a)

$$n\ln(2+\tfrac{1}{n}) - n\ln 2 = n\left[\ln(2+\tfrac{1}{n}) - \ln 2\right] = n\ln\left(\frac{2+\frac{1}{n}}{2}\right)$$

$$= n\ln\left(1+\frac{\frac{1}{2}}{n}\right) = \ln\left[\left(1+\frac{\frac{1}{2}}{n}\right)^n\right]$$

$$\lim_{n\to\infty}\left(1+\frac{\frac{1}{2}}{n}\right)^n = e^{\frac{1}{2}}$$

$$\Rightarrow \lim_{n\to\infty}\left[n\ln(2+\tfrac{1}{n}) - n\ln 2\right] = \ln(e^{\frac{1}{2}}) = \frac{1}{2}$$

b)

$$\lim_{\substack{x\to 0\\ x>0}} \frac{\ln(1+x)}{x} \overset{x=\frac{1}{n}}{\underset{n=\frac{1}{x}}{=}} \lim_{n\to\infty}\left[n\ln(1+\tfrac{1}{n})\right]$$

$$= \lim_{n\to\infty} \ln\left[(1+\tfrac{1}{n})^n\right] = \ln e = 1$$

$$\lim_{\substack{x\to 0\\ x<0}} \frac{\ln(1+x)}{x} \overset{x=-\frac{1}{n}}{=} \lim_{n\to\infty}\left[-n\ln(1-\tfrac{1}{n})\right]$$

$$= -\lim \ln\left[\left(1+\tfrac{-1}{n}\right)^n\right] = -\ln(e^{-1}) = 1$$

$$\Rightarrow \lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$$

2.9

$$\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1 \qquad \text{s. Aufgabe 2.8 b)}$$

Für die Funktion $f(x) = \begin{cases} \frac{\ln(1+x)}{x} & \text{für } x \in]-1,\infty[\setminus\{0\} \\ 1 & \text{für } x = 0 \end{cases}$

gilt $\lim\limits_{x\to 0} f(x) = 1 = f(0)$

$f(x)$ stetig an der Stelle $x = 0$

2.10 a)

$$\lim_{\substack{x\to 0\\ x<0}} f(x) = \lim_{\substack{x\to 0\\ x<0}} \arctan\frac{1}{x} = -\frac{\pi}{2}$$

$\Rightarrow f(x)$ nicht stetig an der Stelle $x = 0$

b)

$$\lim_{\substack{x\to 0\\ x<0}} f(x) = \lim_{\substack{x\to 0\\ x<0}} \tanh\frac{1}{x} = -1$$

$\Rightarrow f(x)$ nicht stetig an der Stelle $x = 0$

c)

$$\lim_{\substack{x\to 0\\ x<0}} f(x) = \lim_{\substack{x\to 0\\ x<0}} (1+x)^{\frac{1}{x}} \overset{x=-\frac{1}{n}}{=} \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{-n}$$

$$= \lim_{n\to\infty} \frac{1}{\left(1+\frac{-1}{n}\right)^n} = \frac{1}{e^{-1}} = e$$

$$\lim_{\substack{x\to 0\\ x>0}} f(x) = \lim_{\substack{x\to 0\\ x>0}} (1+x)^{\frac{1}{x}} \overset{x=\frac{1}{n}}{=} \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$$

$\lim\limits_{x\to 0} f(x) = e = f(0) \Rightarrow f(x)$ stetig an der Stelle $x = 0$

d)

$$\lim_{\substack{x\to 0\\ x<0}} f(x) = \lim_{\substack{x\to 0\\ x<0}} (1+|x|)^{\frac{1}{x}} \overset{x=-\frac{1}{n}}{=} \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{-n}$$

$$= \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} = \frac{1}{e}$$

$\Rightarrow f(x)$ nicht stetig an der Stelle $x = 0$

e) Für die Folge (x_n) mit $x_n = \frac{1}{(2n+1)\pi}$ gilt

$$\lim_{n\to\infty} x_n = 0 \quad \text{und} \quad \lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} \cos\left(\frac{1}{x_n}\right)$$

$$= \lim_{n\to\infty} \cos\left((2n+1)\pi\right) = -1$$

Für die Folge (x_n) mit $x_n = \frac{1}{2n\pi}$ gilt

$$\lim_{n\to\infty} x_n = 0 \quad \text{und} \quad \lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} \cos\left(\frac{1}{x_n}\right)$$

$$= \lim_{n\to\infty} \cos(2n\pi) = 1$$

$\Rightarrow \lim_{x\to 0} f(x)$ existiert nicht $\Rightarrow f(x)$ nicht stetig an der Stelle $x = 0$

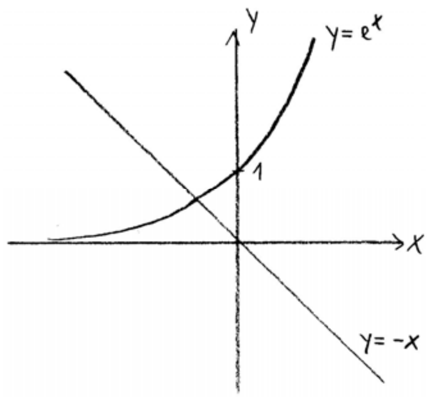
f) Für jede Folge (x_n) mit $x_n \neq 0$ und $\lim_{n\to\infty} x_n = 0$

gilt $\cos \frac{1}{x_n} \in [-1,1]$ und damit

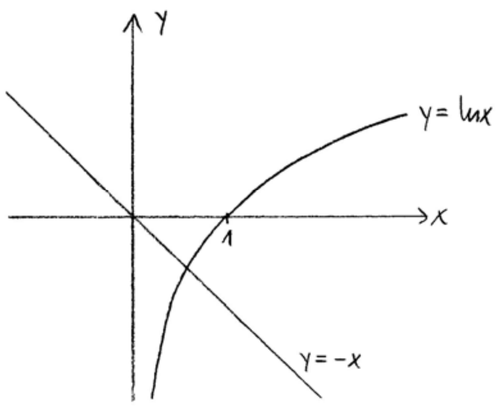
$$\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} \left(x_n \cos \frac{1}{x_n}\right) = 0 = f(0)$$

$\Rightarrow f(x)$ stetig an der Stelle $x = 0$

2.11 a)



Die Gleichung $e^x = -x$ hat genau eine Lösung b)



Die Gleichung $\ln x = -x$ hat genau eine Lösung

2.12 a) $\cos(2x) = 2\sin^2 x$

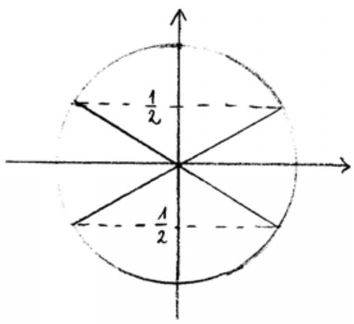
Formelsammlung: $\cos(2x) = 1 - 2\sin^2 x$

$1 - 2\sin^2 x = 2\sin^2 x \qquad 1 = 4\sin^2 x \qquad \sin^2 x = \frac{1}{4}$

$\sin x = \pm \frac{1}{2}$

Vier Lösungen $\in [0, 2\pi]$

$\frac{\pi}{6}, \frac{5}{6}\pi, \frac{7}{6}\pi, \frac{11}{6}\pi$



Alle Lösungen:

$\frac{\pi}{6} + k\pi \quad k \in \mathbb{Z} \qquad \frac{5}{6}\pi + k\pi \quad k \in \mathbb{Z}$

b) $\sin(2x) = 2\cos^2 x$

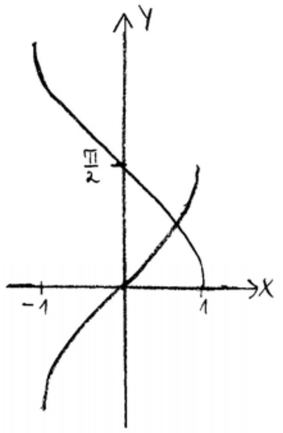
Formelsammlung: $\sin(2x) = 2\sin x \cos x$

$2\sin x \cos x = 2\cos^2 x \qquad \cos^2 x - \sin x \cos x = 0$

$\cos x(\cos x - \sin x) = 0$

$\cos x = 0$ für $x = \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$

$\cos x = \sin x$ für $x = \frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}$ Lösungen c)



$\arccos x = \arcsin x$

genau eine Lösung $\in [0,1]$

$\cos(\arccos x) = \cos(\arcsin x)$

$x = \cos\left(\arccos\sqrt{1-x^2}\right) = \sqrt{1-x^2}$

$x = \sqrt{1-x^2} \qquad x^2 = 1-x^2 \qquad 2x^2 = 1$

$x = \frac{1}{\sqrt{2}}$

d)

$2e^x - 3e^{-3x} = 0 \qquad 2e^x = 3e^{-3x} \qquad \ln(2e^x) = \ln(3e^{-3x})$

$\ln 2 + x = \ln 3 - 3x \qquad 4x = \ln 3 - \ln 2 = \ln\frac{3}{2} \qquad x = \frac{1}{4}\ln\frac{3}{2}$

e)

$4^{2x-3} - 5^{-4x+2} = 0 \qquad 4^{2x-3} = 5^{-4x+2}$

$\ln\left(4^{2x-3}\right) = \ln\left(5^{-4x+2}\right) \qquad (2x-3)\ln 4 = (-4x+2)\ln 5$

$2x\ln 4 + 4x\ln 5 = 2\ln 5 + 3\ln 4$

$2x(\ln 4 + 2\ln 5) = 2\ln 5 + 3\ln 4$

$x = \frac{2\ln 5 + 3\ln 4}{2(\ln 4 + 2\ln 5)} = \frac{\ln 5 + 3\ln 2}{2(\ln 2 + \ln 5)}$

f)

$$\frac{1}{1+e^{-x}} = \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$e^x + e^{-x} = (e^x - e^{-x})(1+e^{-x}) = e^x + 1 - e^{-x} - e^{-2x}$$

$$1 - 2e^{-x} - e^{-2x} = 0 \quad \Big| \cdot e^{2x}$$

$$e^{2x} - 2e^x - 1 = 0 \qquad (e^x)^2 - 2e^x - 1 = 0 \qquad e^x = u$$

$$u^2 - 2u - 1 = 0 \qquad u = \tfrac{1}{2}(2 + \sqrt{8}) = 1 + \sqrt{2}$$

$$x = \ln u = \ln(1+\sqrt{2})$$

oder

$$1 - 2e^{-x} - e^{-2x} = 0 \quad \Big| \cdot e^x$$

$$e^x - 2 - e^{-x} = 0 \qquad e^x - e^{-x} = 2$$

$$2\sinh x = 2 \qquad \sinh x = 1 \qquad x = \operatorname{arsinh} 1$$

g)

$$1 + e^x = \frac{1}{1 - e^{-x}} \qquad (1+e^x)(1-e^{-x}) = 1$$

$$1 + e^x - e^{-x} - 1 = 1$$

$$e^x - e^{-x} = 1$$

$$e^x - e^{-x} - 1 = 0 \quad \Big| \cdot e^x$$

$$(e^x)^2 - e^x - 1 = 0 \qquad e^x = u$$

$$u^2 - u - 1 = 0$$

$$u = \tfrac{1}{2}(1+\sqrt{5}) \qquad x = \ln u = \ln\left(\tfrac{1}{2}(1+\sqrt{5})\right)$$

oder

$$e^x - e^{-x} = 1 \qquad 2\sinh x = 1 \qquad \sinh x = \tfrac{1}{2}$$

$$x = \operatorname{arsinh} \tfrac{1}{2}$$

2.13

$$\tanh\left(\tfrac{1}{2}\ln(1+x^2)\right) = \tanh\left(\ln\left((1+x^2)^{\frac{1}{2}}\right)\right) = \tanh\left(\ln\sqrt{1+x^2}\right)$$

$$= \frac{e^{\ln\sqrt{1+x^2}} - e^{-\ln\sqrt{1+x^2}}}{e^{\ln\sqrt{1+x^2}} + e^{-\ln\sqrt{1+x^2}}} = \frac{\sqrt{1+x^2} - \frac{1}{\sqrt{1+x^2}}}{\sqrt{1+x^2} + \frac{1}{\sqrt{1+x^2}}}$$

$$= \frac{\sqrt{1+x^2}\,\sqrt{1+x^2} - 1}{\sqrt{1+x^2}\,\sqrt{1+x^2} + 1} = \frac{1+x^2-1}{1+x^2+1} = \frac{x^2}{x^2+2}$$

2.14

$$y = \tfrac{1}{2}(e^x - e^{-x}) \qquad e^x - e^{-x} = 2y \quad |\cdot e^x$$

$$(e^x)^2 - 1 = 2y\,e^x \qquad (e^x)^2 - 2y\,e^x - 1 = 0 \qquad e^x = u$$

$$u^2 - 2yu - 1 = 0 \qquad u = \tfrac{1}{2}\left(2y + \sqrt{4y^2+4}\right) = \tfrac{1}{2}\left(2y + 2\sqrt{y^2+1}\right)$$

$$u = y + \sqrt{y^2+1} \qquad x = \ln u = \ln\left(y + \sqrt{y^2+1}\right) = f^{-1}(y)$$

$$f^{-1}(x) = \ln\left(x + \sqrt{x^2+1}\right)$$

2.15

$$f(t_0+t) = f(t_0)\,e^{\lambda t}$$

$$f(t_0)\,e^{\lambda t} = \tfrac{1}{4} f(t_0) \qquad e^{\lambda t} = \tfrac{1}{4} \qquad \lambda t = \ln\tfrac{1}{4} = -\ln 4$$

$$t = \frac{-\ln 4}{\lambda} = \frac{-\ln 4}{-0{,}0033\,\frac{1}{s}} \approx 420\,s = 7\,\text{min}$$

2.16

$$f(t) = A\,e^{-\delta t}\sin\left(\tfrac{2\pi}{T}t + \alpha\right)$$

$$f(t+T) = A\,e^{-\delta(t+T)}\sin\left(\tfrac{2\pi}{T}(t+T) + \alpha\right)$$

$$= A\,e^{-\delta t - \delta T}\sin\left(\tfrac{2\pi}{t} + 2\pi + \alpha\right)$$

$$= A\,e^{-\delta t}\,e^{-\delta T}\sin\left(\tfrac{2\pi}{T} + \alpha\right)$$

$$\frac{f(t)}{f(t+T)} = \frac{A\,e^{-\delta t}\sin\left(\frac{2\pi}{T} + \alpha\right)}{A\,e^{-\delta t}\,e^{-\delta T}\sin\left(\frac{2\pi}{T} + \alpha\right)} = \frac{1}{e^{-\delta T}} = e^{\delta T}$$

$$T = 2{,}75\,s \quad \delta = 0{,}62\,\tfrac{1}{s} \qquad \frac{f(t)}{f(t+T)} = e^{\delta T} \approx 5{,}5$$

2.17

$$v = u \tanh\left(\frac{t}{\tau}\right) \qquad s = u\tau \ln\left(\cosh\left(\frac{t}{\tau}\right)\right)$$

$$\ln\left(\cosh\left(\frac{t}{\tau}\right)\right) = \frac{s}{u\tau} \qquad \cosh\left(\frac{t}{\tau}\right) = e^{\frac{s}{u\tau}}$$

$$\frac{t}{\tau} = \operatorname{arcosh}\left(e^{\frac{s}{u\tau}}\right)$$

$$v = u \tanh\left(\operatorname{arcosh}\left(e^{\frac{s}{u\tau}}\right)\right)$$

Formelsammlung: $\operatorname{arcosh} x = \operatorname{artanh} \frac{\sqrt{x^2-1}}{x}$

$$\operatorname{arcosh}\left(e^{\frac{s}{u\tau}}\right) = \operatorname{artanh}\left(\frac{\sqrt{\left(e^{\frac{s}{u\tau}}\right)^2 - 1}}{e^{\frac{s}{u\tau}}}\right)$$

$$\frac{\sqrt{\left(e^{\frac{s}{u\tau}}\right)^2 - 1}}{e^{\frac{s}{u\tau}}} = \sqrt{\frac{\left(e^{\frac{s}{u\tau}}\right)^2 - 1}{\left(e^{\frac{s}{u\tau}}\right)^2}} = \sqrt{1 - \frac{1}{\left(e^{\frac{s}{u\tau}}\right)^2}}$$

$$= \sqrt{1 - e^{-\frac{2}{u\tau}\cdot s}}$$

$$\operatorname{arcosh}\left(e^{\frac{s}{u\tau}}\right) = \operatorname{artanh}\sqrt{1 - e^{-\frac{2}{u\tau}s}}$$

$$v = u \tanh \operatorname{artanh}\sqrt{1 - e^{-\frac{2}{u\tau}s}} = u\sqrt{1 - e^{-\frac{2}{u\tau}s}}$$

2.18

$$p = p_0\left(1 - \frac{\varkappa-1}{\varkappa}\frac{\rho_0}{p_0} g h\right)^{\frac{\varkappa}{\varkappa-1}}$$

$$= p_0\left(1 + \frac{-\frac{\rho_0}{p_0} g h}{\frac{\varkappa}{\varkappa-1}}\right)^{\frac{\varkappa}{\varkappa-1}}$$

$$= p_0\left(1 + \frac{-\frac{\rho_0}{p_0} g h}{r}\right)^{r} \qquad \text{mit } r = \frac{\varkappa}{\varkappa-1}$$

$$\lim_{\varkappa \to 1}\left(p_0\left(1 - \frac{\varkappa-1}{\varkappa}\frac{\rho_0}{p_0} g h\right)^{\frac{\varkappa}{\varkappa-1}}\right)$$

$$= \lim_{r\to\infty}\left(p_0\left(1 + \frac{-\frac{\rho_0}{p_0} g h}{r}\right)^{r}\right) = p_0 e^{-\frac{\rho_0}{p_0} g h}$$